## Cambridge International AS \& A Level

## MATHEMATICS <br> 9709/12 <br> Paper 1 Pure Mathematics 1 <br> October/November 2021 <br> MARK SCHEME

Maximum Mark: 75
Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers

Cambridge International will not enter into discussions about these mark schemes

Cambridge International is publishing the mark schemes for the October/November 2021 series for most Cambridge IGCSE ${ }^{\text {TM }}$, Cambridge International A and AS Level components and some Cambridge O Level components.

## Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

## GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.


## GENERIC MARKING PRINCIPLE 2:

Marks awarded are always whole marks (not half marks, or other fractions).

## GENERIC MARKING PRINCIPLE 3:

Marks must be awarded positively:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.


## GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

## GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

## GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles
1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5 Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

## Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

## Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.
DM or DB When a part of a question has two or more 'method' steps, the $M$ marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.


## Abbreviations

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO Correct Working Only
ISW Ignore Subsequent Working
SOI Seen Or Implied
SC Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

WWW Without Wrong Working

AWRT Answer Which Rounds To

| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 1 | $2 \cos ^{2} \theta-7 \cos \theta+3[=0]$ | M1 | Forming a 3-term quadratic expression with all terms on the same side or correctly set up prior to completing the square. Allow $\pm$ sign errors. |
|  | $(2 \cos \theta-1)(\cos \theta-3)=0$ | DM1 | Solving their 3-term quadratic using factorisation, formula or completing the square. |
|  | [ $\cos \theta=\frac{1}{2}$ or $\cos \theta=3$ leading to] $\theta=-60^{\circ}$ or $\theta=60^{\circ}$ | A1 |  |
|  | $\theta=-60^{\circ}$ and $\theta=60^{\circ}$ | A1 FT | FT for $\pm$ same answer between $0^{\circ}$ and $90^{\circ}$ or 0 and $\frac{\pi}{2}$. $\pm \frac{\pi}{3}$ or $\pm 1.05$ AWRT scores maximum M1M1A0A1FT. <br> Special case: If M1 DM0 scored then SC B1 for $\theta=-60^{\circ}$ or $\theta=60^{\circ}$, and SC B1 FT can be awarded for $\pm\left(\right.$ their $\left.60^{\circ}\right)$. |
|  |  | 4 |  |


| Question | Answer | Marks | Guidance |
| :---: | :--- | ---: | ---: |
| $2($ a) | Stretch with [scale factor] either $\pm 2$ or $\pm \frac{1}{2}$ | B1 |  |
|  | Scale factor $\frac{1}{2}$ in the $x$-direction | B1 |  |
|  | Translation $\binom{0}{-3}$ or translation of 3 units in negative $y$-direction | B1 |  |
|  |  | B1 B1 | B1 for each correct co-ordinate. |
|  | $(10,9)$ | $\mathbf{2}$ |  |
|  |  | $\mathbf{3}$ |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 3(a) | $\mathrm{f}(5)=[2]$ and $\mathrm{f}($ their 2$)=[5]$ OR ff $(5)=\left[\frac{2+3}{2-1}\right]$ OR $\frac{\frac{x+3}{x-1}+3}{\frac{x+3}{x-1}-1}$ and an attempt to substitute $x=5$. | M1 | Clear evidence of applying f twice with $x=5$. |
|  | 5 | A1 |  |
|  |  | 2 |  |


| Question | Answer | Marks | Guidance |
| :---: | :--- | :--- | :--- |
| $3(\mathrm{~b})$ | $\frac{x+3}{x-1}=y \Rightarrow x+3=x y-y$ OR $\frac{y+3}{y-1}=x \Rightarrow y+3=x y-x$ | $* \mathbf{M 1}$ | Setting $\mathrm{f}(x)=y$ or swapping $x$ and $y$, clearing of fractions and <br> expanding brackets. Allow $\pm$ sign errors. |
|  | $x y-x=y+3 \Rightarrow x=\frac{y+3}{y-1}$ OE OR $y+3=x y-x \Rightarrow y=\left[\frac{x+3}{x-1}\right]$ OE | DM1 | Finding $x$ or $y=$. Allow sign errors. |
|  | $\left[\mathrm{f}^{-1}(x)\right.$ or $\left.y\right]=\frac{\boldsymbol{x}+3}{\boldsymbol{x}-1}$ | A1 | OE e.g. $1+\frac{4}{x-1}$ etc. Must be a function of $x$, cannot be $x=$. |
|  |  | $\mathbf{3}$ |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 4 | 8 | *B1 | For $(3 x+2)^{-1}$ |
|  | $(3 x+2)$ | DB1 | $\text { For }-\frac{8}{3}$ |
|  | $5 \frac{2}{3}=-\frac{\frac{8}{3}}{(3 \times 2+2)}+c$ | M1 | Substituting $\left(2,5 \frac{2}{3}\right)$ into their integrated expression defined by power $=-1$, or dividing by their power. $+c$ needed |
|  | $y=-\frac{8}{3(3 x+2)}+6$ | A1 | OE e.g. $y=-\frac{8}{3}(3 x+2)^{-1}+6$ |
|  |  | 4 |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 5(a) | $\begin{aligned} & {\left[\left(3^{\text {rd }} \text { term }-1^{\text {st }} \text { term }\right)=\left(5^{\text {th }} \text { term }-3^{\text {rd }} \text { term }\right) \text { leading to } \ldots\right]} \\ & -6 \sqrt{3} \sin x-2 \cos x=10 \cos x+6 \sqrt{3} \sin x \\ & {[\text { leading to }-12 \sqrt{3} \sin x=12 \cos x]} \end{aligned}$ <br> OR <br> $\left[\left(1^{\text {st }}\right.\right.$ term $+5^{\text {th }}$ term $)=2 \times 3^{\text {rd }}$ term leading to... $] 12 \cos x=-12 \sqrt{3} \sin x$ | *M1 | OE. From the given terms, obtain 2 expressions relating to the common difference of the arithmetic progression, attempt to solve them simultaneously and achieve an equation just involving $\sin x$ and $\cos x$. |
|  | Elimination of $\sin x$ and $\cos x$ to give an expression in $\tan x$ $\left[\tan x=-\frac{1}{\sqrt{3}}\right]$ | DM1 | For use of $\frac{\sin x}{\cos x}=\tan x$ |
|  | $[x=] \frac{5 \pi}{6}$ only | A1 | CAO. Must be exact. |
|  |  | 3 |  |
| 5(b) | $d=2 \cos x$ or $d=2 \cos ($ their $x)$ | B1 FT | Or an equivalent expression involving $\sin x$ and $\cos x$ e.g. $-3 \sqrt{3} \sin ($ their $x)-\cos ($ their $x)[=-\sqrt{3}]$ <br> FT for their $x$ from (a) only. If not $\pm \sqrt{3}$, must see unevaluated form. |
|  | $\begin{aligned} & \mathrm{S}_{25}=\frac{25}{2}(2 \times(2 \cos (\text { their } x))+(25-1) \times(\text { their } d)) \\ & {[=12.5(2 \times(-\sqrt{3})+24(-\sqrt{3}))]} \end{aligned}$ | M1 | Using the correct sum formula with $\frac{25}{2},(25-1)$ and with $a$ replaced by either $2(\cos ($ their $x))$ or $\pm \sqrt{3}$ and $d$ replaced by either $2(\cos ($ their $x))$ or $\pm \sqrt{3}$. |
|  | $-325 \sqrt{3}$ | A1 | Must be exact. |
|  |  | 3 |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 6 | $a r=54 \text { and } \frac{a \text { or } \text { their } a}{1-r}=243$ | B1 | SOI |
|  | $\begin{aligned} & \frac{54}{r}=243(1-r) \text { leading to } 243 r^{2}-243 r+54[=0]\left[9 r^{2}-9 r+2=0\right] \\ & \text { OR } a^{2}-243 a+13122[=0] \end{aligned}$ | *M1 | Forming a 3-term quadratic expression in $r$ or $a$ using their 2 nd term and $\mathrm{S}_{\infty}$. Allow $\pm$ sign errors. |
|  | $k(3 r-2)(3 r-1)[=0]$ OR $(a-81)(a-162)[=0]$ | DM1 | Solving their 3-term quadratic using factorisation, formula or completing the square. If factorising, factors must expand to give $\pm$ their coefficient of $r^{2}$. |
|  | $54 \div\left(\right.$ their $\left.\frac{2}{3}\right)=a$ OR $54 \div($ their 81$)=r$ | DM1 | May be implied by final answer. |
|  | Tenth term $=\frac{512}{243}\left[\right.$ OR $81 \times\left(\frac{2}{3}\right)^{9}$ OR $\left.54 \times\left(\frac{2}{3}\right)^{8}\right]$ | A1 | OE. Must be exact. <br> Special case: If B1M1DM0DM1 scored then SC B1 can be awarded for the correct final answer. |
|  |  | 5 |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 7(a) | EITHER <br> By using trigonometry: $B \hat{A} C=0.6435 \ldots$ and $A \hat{B} C=\frac{\pi-0.6435}{2}$ OR <br> By Pythagoras: $A P=12 \Rightarrow B P=3$ so $\tan A \hat{B} C=\frac{9}{3}$ <br> OR <br> Using $\triangle P B C$ and either the sine or cosine rule $\sin A \hat{B} C=\frac{3}{\sqrt{10}}$ or $\cos A \hat{B} C=\frac{\sqrt{10}}{10}$ | M1 | $\frac{3}{\sqrt{10}}=0.9486 \ldots \frac{\sqrt{10}}{10}=0.3162 \ldots$ |
|  | $\begin{aligned} & A \hat{B} C=\frac{\pi-0.6435}{2} \text { or } \tan ^{-1} \frac{9}{3} \text { or } \sin ^{-1} \frac{3}{\sqrt{10}} \text { or } \cos ^{-1} \frac{\sqrt{10}}{10} \text { or } \\ & 1.249(04 \ldots) \text { or } 71.56^{\circ}=1.25 \text { radians }(3 \mathrm{sf}) \end{aligned}$ | A1 | AG. Final answer must be 1.25 , more accurate value $1.24904 \ldots$ with no rounding to 3 sf seen as the final answer gets M1A0. <br> If decimals are used all values must be given to at least 4 sf for A1. |
|  |  | 2 |  |
| 7(b) | $B C=\sqrt{(\text { their } 3)^{2}+9^{2}} \text { or } \frac{9}{\sin 1.25}[=\sqrt{90}, 3 \sqrt{10} \text { or } 9.48697 \ldots]$ | M1 | Using correct method(s) to find $B C$. |
|  | Area of sector $=\frac{1}{2} \times(\text { their } B C)^{2} \times \tan ^{-1} 3[=56.207$ or 56.25$]$ | M1 | Using $\tan ^{-1} 3$ or 1.25 and their $B C$, but not 9 or 15 , in correct area of sector formula. |
|  | Area of triangle $P B C=13.4$ to 13.6 or $\frac{1}{2} \times 9 \times 3$ | B1 |  |
|  | [ $\mathrm{Area}=(56.207$ or 56.25$)-$ their $13.5=$ ] 42.7 or 42.8 | A1 | AWRT |
|  |  | 4 |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 8(a) | Terms required for $x^{2}:-5 \times 2^{4} \times a x+10 \times 2^{3} \times a^{2} x^{2}\left[=-80 a x+80 a^{2} x^{2}\right]$ | B1 | Can be seen as part of an expansion or in correct products. |
|  | $2 \times( \pm$ their coefficient of $x)+4 \times\left( \pm\right.$ their coefficient of $\left.x^{2}\right)$ | *M1 |  |
|  | $\begin{aligned} & x^{2} \text { coefficient is } 320 a^{2}-160 a=-15 \\ & \Rightarrow 64 a^{2}-32 a+3 \Rightarrow(8 a-3)(8 a-1) \end{aligned}$ | DM1 | Forming a 3 -term quadratic in $a$, with all terms on the same side or correctly setting up prior to completing the square and solving using factorisation, formula or completing the square. If factorising, factors must expand to give their coefficient of $a^{2}$ 。 |
|  | $a=\frac{1}{8}$ or $a=\frac{3}{8}$ | A1 | OE. <br> Special case: If DM0 for solving quadratic, SC B1 can be awarded for correct final answers. |
|  |  | 4 |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 8(b) | $320 a^{2}-160 a=k \Rightarrow 320 a^{2}-160 a-k[=0]$ | M1 | Forming a 3-term quadratic in $a$ with all terms on the same side. Allow $\pm$ sign errors. |
|  | Their $b^{2}-4 a c[=0],\left[160^{2}-4 \times 320 \times(-k)=0\right]$ | M1 | Any use of discriminant on a 3-term quadratic. |
|  | $k=-20$ | A1 |  |
|  | $a=\frac{1}{4}$ | B1 | Condone $a=\frac{1}{4}$ from $k=20$. |
|  | Alternative method for question 8(b) |  |  |
|  | $320 a^{2}-160 a=k$ and divide by $320\left[a^{2}-\frac{a}{2}=\frac{k}{320}\right]$ | M1 | Allow $\pm$ sign errors. |
|  | Attempt to complete the square $\left[\left(a-\frac{1}{4}\right)^{2}-\frac{1}{16}=\frac{k}{320}\right]$ | M1 | Must have $\left(a-\frac{1}{4}\right)^{2}$ |
|  | $a=\frac{1}{4}$ | A1 |  |
|  | $k=-20$ | B1 |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 8(b) cont'd | Alternative method for question 8(b) |  |  |
|  | $320 a^{2}-160 a=k$ and attempt to differentiate LHS [640a-160] | M1 | Allow $\pm$ sign errors. |
|  | Setting their $(640 a-160)=0$ and attempt to solve. | M1 |  |
|  | $a=\frac{1}{4}$ | A1 |  |
|  | $k=-20$ | B1 |  |
|  |  | 4 |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 9(a) | $\left[\frac{\mathrm{d} V}{\mathrm{~d} r}=\right] \frac{9}{2}\left(r-\frac{1}{2}\right)^{2}$ | B1 | OE. Accept unsimplified. |
|  | $\frac{\mathrm{d} r}{\mathrm{~d} t}=\frac{\mathrm{d} r}{\mathrm{~d} V} \times \frac{\mathrm{d} V}{\mathrm{~d} t}=\frac{1.5}{\text { their } \frac{\mathrm{d} V}{\mathrm{~d} r}}\left[=\frac{1.5}{\frac{9}{2}\left(5.5-\frac{1}{2}\right)^{2}}=\frac{1.5}{112.5}\right]$ | M1 | Correct use of chain rule with 1.5 , their differentiated expression for $\frac{\mathrm{d} V}{\mathrm{~d} r}$ and using $r=5.5$. |
|  | 0.0133 or $\frac{3}{225}$ or $\frac{1}{75}$ [ metres per second] | A1 |  |
|  |  | 3 |  |
| 9(b) | $\frac{\mathrm{d} V}{\mathrm{~d} r}$ or their $\frac{\mathrm{d} V}{\mathrm{~d} r}=\frac{1.5}{0.1}$ or 15 OR $0.1=\frac{1.5}{\text { their } \frac{\mathrm{d} V}{\mathrm{~d} r}}\left[=\frac{2 \times 1.5}{9\left(r-\frac{1}{2}\right)^{2}}\right.$ OE $]$ | B1 FT | Correct statement involving $\frac{\mathrm{d} V}{\mathrm{~d} r}$ or their $\frac{\mathrm{d} V}{\mathrm{~d} r}, 1.5$ and 0.1 . |
|  | $\left[\frac{9}{2}\left(r-\frac{1}{2}\right)^{2}=15 \Rightarrow\right] r=\frac{1}{2}+\sqrt{\frac{10}{3}}$ | B1 | OE e.g. AWRT 2.3 <br> Can be implied by correct volume. |
|  | [Volume $=$ ] 8.13 AWRT | B1 | OE e.g. $\frac{-3+5 \sqrt{30}}{3}$. CAO. |
|  |  | 3 |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 10(a) | $\left[\mathrm{f}^{\prime}(x)=\right] 2 x-\frac{k}{x^{2}}$ | B1 |  |
|  | $\mathrm{f}^{\prime}(2)=0\left[2 \times 2-\frac{k}{2^{2}}=0\right] \Rightarrow k=\ldots$ | M1 | Setting their 2 -term $\mathrm{f}^{\prime}(2)=0$, at least one term correct and attempting to solve as far as $k=$. |
|  | $k=16$ | A1 |  |
|  |  | 3 |  |
| 10(b) | $\mathrm{f}^{\prime \prime}(2)=$ e.g. $2+\frac{2 k}{2^{3}}$ | M1 | Evaluate a two term $\mathrm{f}^{\prime \prime}(2)$ with at least one term correct. Or other valid method. |
|  | $\left[2+\frac{2 k}{2^{3}}\right]>0 \Rightarrow$ minimum or $=6 \Rightarrow$ minimum | A1 FT | WWW. FT on positive $k$ value. |
|  |  | 2 |  |
| 10(c) | When $x=2, \mathrm{f}(x)=14$ | B1 | SOI |
|  | [Range is or $y$ or $\mathrm{f}(x)] \geqslant$ their $\mathrm{f}(2)$ | B1 FT | Not $x \geqslant$ their $\mathrm{f}(2)$ |
|  |  | 2 |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 11(a) | $\frac{\mathrm{d} y}{\mathrm{~d} x}=\frac{1}{2}+\frac{1}{3(x-2)^{\frac{4}{3}}}$ | B1 | OE. Allow unsimplified. |
|  | Attempt at evaluating their $\frac{\mathrm{d} y}{\mathrm{~d} x}$ at $x=3\left[\frac{1}{2}+\frac{1}{3(3-2)^{\frac{4}{3}}}=\frac{5}{6}\right]$ | *M1 | Substituting $x=3$ into their differentiated expression defined by one of 3 original terms with correct power of $x$. |
|  | $\text { Gradient of normal }=\frac{-1}{\text { their } \frac{d y}{d x}}\left[=-\frac{6}{5}\right]$ | *DM1 | Negative reciprocal of their evaluated $\frac{\mathrm{d} y}{\mathrm{~d} x}$. |
|  | Equation of normal $y-\frac{6}{5}=($ their normal gradient $)(x-3)$ $\left[y=-\frac{6}{5} x+4.8 \Rightarrow 5 y=-6 x+24\right]$ | DM1 | Using their normal gradient and $A$ in the equation of a straight line. <br> Dependent on *M1 and *DM1. |
|  | [When $y=0,] x=4$ | A1 | or (4, 0) |
|  |  | 5 |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 11(b) | $\text { Area under curve }=\int\left(\frac{1}{2} x+\frac{7}{10}-\frac{1}{(x-2)^{\frac{1}{3}}}\right)[\mathrm{d} x]$ | M1 | For intention to integrate the curve (no need for limits). Condone inclusion of $\pi$ for this mark. |
|  | $\frac{1}{4} x^{2}+\frac{7}{10} x-\frac{3(x-2)^{\frac{2}{3}}}{2}$ | A1 | For correct integral. Allow unsimplified. Condone inclusion of $\pi$ for this mark. |
|  | $\left(\frac{9}{4}+2.1-\frac{3}{2}\right)-\left(\frac{6.25}{4}+1.75-\frac{3 \times 0.5^{\frac{2}{3}}}{2}\right)$ | M1 | Clear substitution of 3 and 2.5 into their integrated expression (with at least one correct term) and subtracting. |
|  | 0.48[24] | A1 | If M1A1M0 scored then SC B1 can be awarded for correct answer. |
|  | [ Area of triangle $=$ ] 0.6 | B1 | OE |
|  | [Total area $=$ ] 1.08 | A1 | Dependent on the first M1 and WWW. |
|  |  | 6 |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 12(a) | Centre is $(3,-2)$ | B1 |  |
|  | $\text { Gradient of radius }=\frac{(\text { their }-2)-4}{(\text { their } 3)-5}[=3]$ | *M1 | Finding gradient using their centre (not (0,0)) and $P(5,4)$. |
|  | Equation of tangent $y-4=-\frac{1}{3}(x-5)$ | DM1 | Using $P$ and the negative reciprocal of their gradient to find the equation of $A B$. |
|  | Sight of $[x=] 17$ and $[y=] \frac{17}{3}$ | A1 |  |
|  | $\left[\right.$ Area $\left.=\frac{1}{2} \times \frac{17}{3} \times 17=\right] \frac{289}{6}$ | A1 | Or $48 \frac{1}{6}$ or AWRT 48.2. |
|  | Alternative method for question 12(a) |  |  |
|  | $2 x+2 y \frac{\mathrm{~d} y}{\mathrm{~d} x}-6+4 \frac{\mathrm{~d} y}{\mathrm{~d} x}=0$ | B1 |  |
|  | At $P: 10+8 \frac{\mathrm{~d} y}{\mathrm{~d} x}-6+4 \frac{\mathrm{~d} y}{\mathrm{~d} x}=0\left[\Rightarrow \frac{\mathrm{~d} y}{\mathrm{~d} x}=-\frac{1}{3}\right]$ | *M1 | Find the gradient using $P(5,4)$ in their implicit differential (with at least one correctly differentiated $y$ term). |
|  | Equation of tangent $y-4=-\frac{1}{3}(x-5)$ | DM1 | Using $P$ and their value for the gradient to find the equation of $A B$. |
|  | Sight of $[x=] 17$ and $[y=] \frac{17}{3}$ | A1 |  |
|  | $\left[\right.$ Area $\left.=\frac{1}{2} \times \frac{17}{3} \times 17=\right] \frac{289}{6}$ | A1 | Or $48 \frac{1}{6}$ or AWRT 48.2. |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| $12(\mathrm{a})$cont'd | Alternative method for question 12(a) |  |  |
|  | $\left[y=-2 \pm\left(40-(x-3)^{2}\right)^{\frac{1}{2}}\right.$ OE leading to $] \frac{\mathrm{d} y}{\mathrm{~d} x}=(3-x)\left(31+6 x-x^{2}\right)^{-\frac{1}{2}}$ | B1 | OE. Correct differentiation of rearranged equation. |
|  | $\frac{\mathrm{d} y}{\mathrm{~d} x}=(3-5)\left(31+6(5)-(5)^{2}\right)^{-\frac{1}{2}}\left[\Rightarrow \frac{\mathrm{~d} y}{\mathrm{~d} x}=-\frac{1}{3}\right]$ | *M1 | Find the gradient using $x=5$ in their differential (with clear use of chain rule). |
|  | Equation of tangent $y-4=-\frac{1}{3}(x-5)$ | DM1 | Using $P$ and their value for the gradient to find the equation of $A B$. |
|  | Sight of $[x=] 17$ and $[y=] \frac{17}{3}$ | A1 |  |
|  | $\left[\right.$ Area $\left.=\frac{1}{2} \times \frac{17}{3} \times 17=\right] \frac{289}{6}$ | A1 | Or $48 \frac{1}{6}$ or AWRT 48.2. |
|  |  | 5 |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 12(b) | Radius of circle $=\sqrt{40}$, | B1 | Or $2 \sqrt{10}$ or 6.32 AWRT or $r^{2}=40$. |
|  | Area of $\triangle C R Q=\frac{1}{2} \times(\text { their } r)^{2} \sin 120\left[=\frac{1}{2} \times 40 \times \frac{\sqrt{3}}{2}\right]$ <br> OR <br> Area of $\triangle C Q X=\frac{1}{2} \times \sqrt{40} \cos 30 \times \sqrt{40} \cos 60$ OE $\left[=\frac{1}{2} \times \sqrt{30} \times \sqrt{10}\right]$ OR <br> Area of circle $-3 \times$ Area of segment $=40 \pi-3 \times\left(40 \frac{\pi}{3}-10 \sqrt{3}\right)$ OR <br> $Q R=\sqrt{120}$ or $2 \sqrt{30}$ and area $=\frac{1}{2} Q R^{2} \sin 60$ | M1 | Using $\frac{1}{2} r^{2} \sin \theta$ with their $r$ and 120 or $60[\times 3]$ <br> Using $\frac{1}{2} \times$ base $\times$ height in a correct right-angled triangle [×6]. <br> Use of cosine rule and area of large triangle |
|  | $30 \sqrt{3}$ | A1 | AWRT 52[.0] implies B1M1A0. |
|  |  | 3 | See diagram for points stated in 'Answer' column. |

